Maharashtra Education Society's

Abasaheb Garware College

(Autonomous)

(Affiliated to Savitribai Phule Pune University)

Two Year M. Sc. Degree Program in Mathematics

(Faculty of Science and Technology)

Syllabi under Autonomy

M. Sc. II (Mathematics)

Choice Based Credit System Syllabus

To be implemented from Academic Year 2023-2024

Title of the Course: M. Sc. (Mathematics)

Preamble:

Taking into consideration the rapid changes in science and technology and new approaches in different areas of Mathematics and related subjects, Board of studies in Mathematics after a thorough discussion with the teachers of Mathematics from Abasaheb Garware College and experts from industry as well as other Academic institutions has prepared the syllabus of M.Sc.II Semester -III and Semester- IV (w. e. f. 2023-24) Mathematics course under the Choice Based Credit System (CBCS). The model curriculum as developed by U. G. C. is used as a guideline for the present syllabus.

Program Outcomes:

1) To maintain updated curriculum.
2) To take care of fast development in the knowledge of Mathematics.
3) To enhance the quality and standards of Mathematics Education.
4) To provide a broad common frame work, for exchange, mobility and free dialogue across the Indian Mathematicians and associated community.
5) To create an aptitude for Mathematics in those students who show a promise for higher studies and creative work in Mathematics.
6) To create confidence in others, for equipping themselves with that part of Mathematics which is needed for various branches of Sciences or Humanities in which they have aptitude for higher studies and original work.

Eligibility:

M. Sc. I [Mathematics] as per SPPU rules.

Structure of the course: M. Sc. Mathematics

Year	Semester	Course Type	Course Code	Course Title	Remark	Credit	No. of Lectures/Practical to be conducted
1	I	Core	PSMT-111	Linear Algebra	Theory	4	60
		Core	PSMT-112	Group Theory	Theory	4	60
		Core	PSMT-113	Advanced Calculus	Theory	4	60
		Core	PSMT-114	Ordinary Differential Equations	Theory	4	60
		Core	PSMT-115	C Programming	Theory	2	30
		Core	PSMTP-115	C Programming	Practical	2	12
	II	Core	PSMT-121	Complex Analysis	Theory	4	60
		Core	PSMT-122	Rings and Modules	Theory	4	60
		Core	PSMT-123	General Topology	Theory	4	60
		Core	PSMT-124	Partial Differential Equations	Theory	4	60
		Core	PSMT-125	Programming with Python	Theory	2	30
		Core	PSMTP-125	Programming with Python	Practical	2	12

Year	Semester	Course Type	Course Code	Course Title	Remark	Credit	No. of Lectures/Practicals to be conducted
2	III	Core	PSMT-231	Measure and Integration	Theory	4	60
		Core	PSMT-232	Integral Equations	Theory	4	60
		Core	PSMT-233	Field theory	Theory	4	60
			Choose any two between PSMTELE-234A and PSMTELE-234F				
		Elective	PSMTELE-234A	Graph Theory	Theory	4	60
		Elective	PSMTELE-234B	Data analytics using Python	Theory	2	30
			PSMTP-234B	Data analytics using Python	Practical	2	12
		Elective	PSMTELE-234C	Classical Mechanics	Theory	4	60
		Elective	PSMTELE-234D	Algebraic topology	Theory	4	60
		Elective	PSMTELE-234E	Number Theory	Theory	4	60
		Elective	PSMTELE-234F	Probability and Statistics	Theory	4	60
	IV	Core	PSMT-241	Numerical Analysis	Theory	4	60
		Core	PSMT-242	Functional analysis	Theory	4	60
		Core	PSMT-243	Differential Geometry	Theory	4	60
			Choose any two between PSMTELE-244A and PSMTELE-244F				
		Elective	PSMTELE-244A	Combinatorics	Theory	4	60
		Elective	PSMTELE-244B	Machine learning using Python	Theory	2	30
			PSMTP-244B	Machine learning using Python	Practical	2	12
		Elective	PSMTELE-244C	Dynamical Systems	Theory	4	60
		Elective	PSMTELE-244D	Commutative Algebra	Theory	4	60
		Elective	PSMTELE-244E	Representation theory of finite groups	Theory	4	60
		Elective	PSMTELE-244F	Coding Theory	Theory	4	60

Additional Credit Courses are as follows:

Year	Semester	Course Code	Mandatory Add-On Credit Course	Credits
I	I	MTHR1-11	Human Rights -I	1
		MTCYS1-11	Cyber Security-I	1
	II	MTHR2-12	Human Rights -II	1
		MTCYS2-12	Cyber Security-II	1
II	III	MTCYS3-23	Cyber Security-III	1
		MTSD1-23	Skill Development-I	1
		MTIC-23	Introduction to Indian Constitution	2
	IV	MTCYS4-24	Cyber Security-IV	1
		MTSD2-24	Skill Development-II	1
			Total Credits	10

Note: Only grade will be given for add-on courses and this will be not be counted for SGPA or CGPA calculations. Student must pass in all add-on courses to get the M. Sc. Mathematics degree.

The study tour is important to learn, network and collaborate with academics, students and industry professionals in Science and Technology, Engineering and Mathematics recruiters. It is also important to know and understand different cultures, easily adapt to new environments, skilfully negotiate business deals.

Therefore, Department of Mathematics should arrange student's study tour in each academic year.

SEMESTER-III

Course Code and Title: PSMT-231: Measure and Integration

Lectures: 60 (Credits- 4)

Course Outcomes:

1. To learn the concept of measure, Lebesgue measure.
2. To learn measurable functions, convergence and integration.
3. To learn L^p spaces and its applications.
4. To learn Fundamental theorem for Lebesgue integrable functions, and functions of bounded variations

Unit I: Measure on the real line **[15 Hours]**

1.1 Lebesgue outer measure
1.2 Measurable sets
1.3 Regularity
1.4 Measurable functions
1.5 Borel and Lebesgue measurability

Unit II: Integration of functions of a real variable **[15 Hours]**

2.1 Integration of non-negative functions
2.2 The general integral
2.3 Integration of series
2.4 Riemann and Lebesgue integrals

Unit III: Differentiation **[20 Hours]**

3.1 The Four Derivatives
3.2 Continuous non-differentiable functions
3.3 Functions of bounded variation
3.4 Lebesgue's Differentiation Theorem
3.5 Differentiation and Integration
3.6 The Lebesgue Set

Unit IV: Inequalities and L^p spaces **[10 Hours]**

4.1 The L^p spaces
4.2 Convex functions
4.3 Jensen's Inequality
4.4 The inequalities of Holder and Minkowski
4.5 Completeness of L^p (m)

Recommended Book:

G de Barra, Measure theory and integration, New Age International Publishers.

Chapters: Chapter 2: 2.1, 2.2, 2.3, 2.4, 2.5,
Chapter 3, Chapter 4, Chapter 6.

Reference Books:

1) Royden, H. L. Real Analysis, 2nd Edition, Macmillan, 1964.
2) Rudin, W. Principles of Mathematical Analysis, Third Edition, McGraw-Hill International Edition, 1976
3) Rudin, W. Real and Complex Analysis, Tata McGraw-Hill, 1974.
4) Paul R. Halmos, Measure theory, Graduate texts in Mathematics, Springer –Verlag
5) S. Kesavan, Measure and Integration, Hindustan Book agency, Springer
6) Robert G. Bartle, The Elements of Integration and Lebesgue Measure, John Wiley and Sons.

Course Code and Title: PSMT-232: Integral Equations

Lectures: 60 (Credits- 4)

Course Outcomes

1. Students are able to understand relationship between Integral Equations and ODEs.
2. To learn Fredholm –Schmidt Theory.
3. To understand variational principle and its applications.

Unit I: Integral equations and Picard's Method [05 Hours]

1.1 Integral equations and their relationship to differential equations
1.2 Picard's method

Unit II: The Nonhomogeneous Linear equations [07 Hours]

2.1 Method of variation of parameters
2.2 Green's functions

Unit III: The Fredholm Alternative [10 Hours]

3.1 A simple case
3.2 Some algebraic preliminaries
3.3 The Fredholm Alternative Theorem

Unit IV: Hilbert-Schmidt Theory [16 Hours]

4.1 Eigenvalues are real and Eigen functions corresponding to distinct eigenvalues are orthogonal
4.2 Orthonormal family of functions and Bessel's inequality
4.3 Some results about eigenvalues deducible from Bessel's inequality
4.4 Description of sets of all eigenvalues and eigenfunctions
4.5 The Expansion Theorem

Unit V: Iterative Methods and Neumann series [10 Hours]

5.1 Power series of an integral operator
5.2 Iterated Kernels

5.3 Neumann series
5.4 Remark on convergence of iterative methods

Unit VI: The Calculus of Variations [12 Hours]

6.1 The fundamental problem
6.2 Some classical examples from mechanics and geometry
6.3 The derivation of Euler's equation for fundamental problem
6.4 The special case F = F (y, y')
6.5 When F contains more dependant functions
6.6 When F contains more independent variables
6.7 Integral constraints
6.8 Non integral constraints
6.9 Varying boundary conditions

Recommended Book:

Peter J Collins, Differential and Integral Equations, Oxford University Press, 2006

Chapters: 1, 4, 8, 9, 10, 11

Reference Books:

1) Kanwal Ram P., Linear Integral Equations, Birkhauser publication 1997
2) Abdul J. Jerri, Introduction to Integral Equations with Applications, Wiley-Interscience; 2nd edition (September 3, 1999)
3) Abul-Majid Wazwaz, A First Course In Integral Equations, World Scientific Publications, 1997.

Course Code and Title: PSMT-233: Field Theory

Lectures: 60 (Credits-4)

Course Outcomes:

1. Students are able to understood field extensions.
2. Students are able to understood Splitting Field of Polynomials.
3. To learn Fundamental Theorem of Galois Theory.
4. To learn applications of Galois Theory.

Unit I: Field Extensions [15 Hours]

1.1 Irreducibility Criteria (revision)
1.2 Basic Theory of Field Extensions
1.3 Algebraic Extensions
1.4 Classical Straightedge and Compass Construction.

Unit II: Splitting Fields [15 Hours]

2.1 Splitting Fields and Algebraic Closures

2.2 Separable and Inseparable Extensions
2.3 Cyclotomic Polynomials and Extensions

Unit III: Galois Theory [15 Hours]

3.1 Automorphism Group of a Field,
3.2 The Fundamental Theorem of Galois Theory
3.3 Finite Fields
3.4 Composite Extension and Simple Extension
3.5 Cyclotomic Extension and Abelian Extensions over Q

Unit IV: Solvability by Radicals [15 Hours]

4.1 Galois Group of Polynomials, Frobenius Automorphism
4.2 Solvable and Radical Extensions
4.3 Insolvability of Quintic Polynomials

Recommended Book:

1. David S. Dummit, Richard M. Foote, Abstract Algebra, second edition, Wiley Student Edition.
   Chapters: 13, 14

Reference Books:

1) Jean-Pierre Escofier, Galois Theory, Graduate Texts in Mathematics, Springer Publication.
2) Joseph Rotman, Galois Theory, Second Edition, Springer Publication.
3) M. Artin, Algebra, second edition, Pearson Publication
4) John m. Howie, Fields and Galois Theory, Springer Publication.

Course Code and Title: PSMTELE-234A: Graph Theory

Lectures: 60 (Credits-4)

Course Outcomes:

1. To understand and apply the fundamental concepts in graph theory
2. Model problems using graphs and to solve these problems algorithmically.
3. Apply theories and concepts to test independent mathematical thinking in problem solving and proof writing.

Unit I: Fundamental concepts [15 Hours]

1.1 What is a Graph?
Graphs as Models; Matrices and Isomorphism; Decomposition and Special Graphs; Degree of a vertex; Counting and Bijections
1.2 Paths, Cycles and Trails: Paths, Cycles, Trails: Connection in Graphs; Bipartite

Graphs; Eulerian Circuits; Hamiltonian Cycles

1.3 Directed Graphs: Definition and Examples; Vertex Degrees; Eulerian Digraphs

Unit II: Trees and Distances [16 Hours]

2.1 Basic Properties: Properties of Trees, Distance in Trees and Graphs

2.2 Spanning Trees and Enumeration: Enumeration of Trees, Spanning Trees in Graphs

2.3 Optimization and Trees: Minimum Spanning Tree, Shortest Paths

Unit III: Matching and Factors [15 Hours]

3.1 Matching and Covers: Maximum Matching's, Hall's Matching Condition, Min-Max Theorem

Unit IV: Connectivity and Paths [12 Hours]

4.1 Cuts and Connectivity: Connectivity; Edge-connectivity

4.2 2-connected Graphs

Unit V: Introduction to Four color theorem and Five color theorem, Planarity [2 Hours]

Recommended Book:

Douglas B. West, Introduction to Graph Theory 2nd Edn, PHI Learning Pvt. Ltd.

Unit I: Section 1.1, 1.2, 1.3 (Counting and Bijections), Section 7.2 (Hamiltonian Cycles) Section 1.4 (Definitions, Vertex Degrees, Eulerian Digraphs)

Unit II: Section 2.1 (Properties of Trees, Distance in Trees and Graphs), Section 2.2 (Enumeration of Trees, Spanning Trees), Section 2.3 (Minimum Spanning Tree, Shortest Path);

Unit II: Section 3.1 (Maximum Matchings, Hall's Matching Condition, Min-Max Theorem)

Unit II: Section 4.1 (Connectivity, Edge-Connectivity), Section 4.2 (2-connected Graphs)

Unit V: Section 6.1 (Euler's Formula without proof), 6.2 (Kuratowski Theorem without proof), Brooks Theorem (without proof), Chromatic polynomial (computation only), Four color theorem (Idea only).

Reference Books:

1) John Clarke and D.A. Holton, A First Look at Graph Theory, Allied Publisher (1991)

2) Harary, Graph Theory, Narosa Publishers, New Delhi (1989)

Course Code and Title: PSMTELE-234B: Data Analytics using Python

Lectures: 30(Credits-2)

Course Outcomes:

1. Learn basic Python required for Data Analytics.

2. Students able to use Visualize data using Python.
3. Study Linear Algebra, Statistics, Probability, Hypothesis and Inference, Gradient Descent using Python.
4. Learn to get data from various sources, work with data.
5. Learn basics of Machine Learning.
6. To understand the concepts of k-nearest Neighbors and Naïve Bayes.

Unit I: **Introduction** **[02 Hours]**

What is Data Science and Data Analytics? Understanding importance, Classes and structure of data

Unit II: Basic Linear Algebra **[09 Hours]**

Vectors, Matrices, Moore-Penrose Inverse of Matrix and its application to solve Linear System of equations, Diagnolisation, SVD

Unit III: Basic Statistics and Probability **[09 Hours]**

Central Tendencies, Dispersion, Correlation,
Conditional Probability, Bayes's Theorem, Random Variables, Continuous Distributions, The Normal Distribution, The Central Limit Theorem, Statistical Hypothesis Testing, Confidence Intervals, Bayesian Inference

Unit IV: Basic Calculus **[10 Hours]**

Derivatives of Matrices, Gradient Descent Method, Lagrange's Method of Undetermined Multipliers, Hamming Metric, Taxicab Metric, Newton Raphson method

Recommended Book:

1) Joel Grus, Data Science from Scratch, First Principles with Python, O'Reilly Publication, Second Edition
Chapter 1 to 13
2) Marc Peter Deisenroth, A. Aldo Faisal, Cheng Soon Ong, Mathematics for Machine learning, Cambridge University Press (2020).

Reference Books:

1) Dirk P. Kroese, Zdravko I. Botev, Thomas Taimre, RadislavVaisman, Data Science and Machine Learning, Mathematical and Statistical Methods
2) Francois Chollet, Deep Learning with Python, Manning Publications
Davy Cielen, Arno D. B. Meysman, Mohmed Ali, Introducing Data Science Big Data, Machine Learning, And more using Python Tools, Manning Publications

Course Code and Title: PSMTP-234B: Practical based on Data Analytics Using Python

(Credits- 2)

One practical 4 hrs. per week (Total 12 Practicals)

Practical I: Python revision

Practical II: Visualizing Data

Matplotlib, Bar Charts, Line Charts, Scatter plots, Vectors and Matrices

Practical III: Importing Data -I

stdin and stdout, Reading Files, The Basics of Text Files, Delimited Files

Practical IV: Importing Data-II

Scraping the Web, HTML and the Parsing Thereof, Using APIs, JSON and XML, Using an Unauthenticated API, Finding APIs

Practical V: Manipulating Data-I

Exploring Your Data, Exploring One-Dimensional Data, Two Dimensions,

Practical VI: Manipulating Data-II

Exploring many Dimensions, Using Named Tuples,

Practical VII: Cleaning and Munging, Rescaling

Practical VIII: Dimensionality Reduction

Practical IX: k-Nearest clustering

The Model, Example: The Iris Dataset, The Curse of Dimensionality, Hierarchical Clustering

Practical X: Naive Bayes

A Really Dumb Spam Filter, A More Sophisticated Spam Filter, Implementation, Testing Our Model, Using Our Model

Practical XI: Linear Regression and Logistic regression

Practical XII: Multiple Regression

Recommended Book:

1) Joel Grus, Data Science from Scratch, First Principles with Python, O'Reilly Publication, Second Edition
   Chapter 1 to 13

Reference Books:

1) Dirk P. Kroese, Zdravko I. Botev, Thomas Taimre, RadislavVaisman, Data Science and Machine Learning, Mathematical and Statistical Methods
2) Francois Chollet, Deep Learning with Python, Manning Publications
   Davy Cielen, Arno D. B. Meysman, Mohmed Ali, Introducing Data Science Big Data, Machine Learning, And more using Python Tools, Manning Publications

Course Code and Title: PSMTELE-234C: Classical Mechanics

Lectures: 60 (Credits-4)

Course Outcomes:

1. To learn about Lagrangian and Hamiltonian formulation of equation of motions.
2. To learn Conservation Theorem and Symmety Properties.
3. Students are able to understood Angular Momentum and Kinetic energy of the motion.
4. To learn Euler's theorem on Motion of a rigid body.

Unit I: Survey of Elementary Principles [15 Hours]

1.1 Mechanics of a Particle
1.2 Mechanics of system of Particles
1.3 Constraints.
1.4 D'Alemberts Principle and Lagrange's Equations
1.5 Velocity dependant potentials and dissipation function
1.6 Simple Applications of Lagrange's Formulation

Unit II: Variational Principle and Lagrange's Equations [15 Hours]

2.1 Hamilton's Principle
2.2 Some techniques of Calculus of variation
2.3 Derivations of Lagrange's Equations from Hamilton's principle
2.4 Extensions of Hamilton's principle to non-holonomic systems
2.5 Advantages of variational Principle Formulation
2.6 Conservation Theorems and Symmetry Properties
2.7 Energy function and the conservation of energy

Unit III: Kinematics of Rigid Body Motion [15 Hours]

3.1 Independent co-ordinates of rigid body
3.2 Orthogonal transformations
3.3 Formal properties of transformation Matrix
3.4 The Euler Angles
3.5 The Cayley Klein Parameters and related quantities
3.6 Euler's theorem on Motion of a rigid body
3.7 Finite Rotations
3.8 Infinitesimal Rotations
3.9 Rate of change of a vector

Unit IV: The Rigid Body Equations of Motion [15 Hours]

4.1 Angular Momentum and Kinetic energy of Motion about a point
4.2 Tensors
4.3 The Inertia Tensor and the Moment of Inertia
4.4 Eigenvalues of the Inertia Tensor and Principal axis Transformations

4.5 Solving Rigid body problems and Euler's Equations of Motion
4.6 Introduction of two body and three body problems.

Recommended Book:

Classical Mechanics (3rd Ed.) by Herbert Goldstein, Charles Poole, John Safko (Pearson Education)
Chapters: Chapter 1, Chapter 2, Chapter 4: 4.1-4.9, Chapter 5: 5.1-5.5

Reference Books:

1) Problems in Classical Mechanics by L. N. Katkar (Narosa Publication)
2) Classical Mechanics by Gupta, Kumar and Sharma (A Pragati Edition).
3) Classical Mechanics by Rana and Joag (McGraw Hill India)

Course Code and Title: PSMTELE-234D: Algebraic Topology

Lectures: 60 (Credits-4)

Course Outcomes:

1. To learn fundamental theorem of algebra, fundamental group of S_n.
2. To learn fundamental groups of wedges of circle, torus and dunce cap.
3. Students learn covering transformations.

Unit I: The Fundamental Group [18 Hours]

1.1 Homotopy of Paths
1.2 The Fundamental Group
1.3 Covering Spaces
1.4 The Fundamental Group of the Circle
1.5 Retractions and Fixed Points
1.6 The Fundamental Theorem of Algebra
1.7 The Borsuk-Ulam Theorem
1.8 Deformation Retracts and Homotopy Type
1.9 The Fundamental Group of S_n

Unit II: The Seifert-van Kampen Theorem [18 Hours]

2.1 Direct Sums of Abelian Groups (only revision)
2.2 Free Products of Groups (only revision)
2.3 Free Groups
2.4 The Seifert-van Kampen Theorem
2.5 The Fundamental Group of a Wedge of Circles
2.6 The Fundamental Groups of the Torus and the Dunce Cap

Unit III: Classification of Covering Spaces [15 Hours]

3.1 Equivalence of Covering Spaces
3.2 The Universal Covering Spaces
3.3 Covering Transformations
3.4 Existence of Covering Spaces

Unit IV: Geometric Simplexes and Complexes [09 Hours]

4.1 Geometrically independent set
4.2 Simplexes
4.2 Orientation of simplexes
4.3 Complexes
4.4 Triangulation
4.5 Simplicial mappings
4.6 Topological dimension
4.7 Brouwer Fixed point theorem

Recommended Book:

James R. Munkres, Topology, Second Edition, Pearson Prentice Hall.
Chapter 9: Sections: 51, 52, 53, 54, 55, 56, 57, 58, 59
Chapter 11: Sections: 67, 68 (Only revision), 69, 70, 71, 73
Chapter 13: Sections: 79, 80, 81, 82

A. Lahiri and B.K. Lahiri, An Introduction to Algebraic Topology, alpha science International Ltd
Chapter 10

Reference Books:

1) Allen Hatcher, Algebraic Topology, Cambridge University Press, 2002.
2) M.A. Armstrong, Basic Topology, Springer International Edition, 2004.
3) J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
4) E. H. Spanier**,** Algebraic Topology, Springer, 1994.
5) A. Lahiri and B.K. Lahiri, An Introduction to Algebraic Topology, alpha science International Ltd.

Course Code and Title: PSMTELE-234E Number Theory

Lectures: 60 (Credits- 4)

Course Outcomes:

1. To introduce number theoretic problems and to different areas of number theory.
2. To understand and apply the fundamental concepts in Number theory
3. To apply skills in calculation and manipulation of problem solving in the following areas: integers, prime numbers, congruences, arithmetic functions, quadratic residues, Diophantine equations.

Unit I: Arithmetical Functions and Dirichlet Multiplication [15 Hours]

1.1 The Mobius function $\mu(n)$
1.2 The Euler Totient function $\phi(n)$ and relation between $\mu(n)$ and $\phi(n)$
1.3 The product formula for $\phi(n)$
1.4 The Dirichlet Product of Arithmetical functions
1.5 Dirichlet Inverses and the Mobius Inversion formula
1.6 Mangoldt function
1.7 Multiplicative functions and Dirichlet Multiplication
1.8 Inverse of completely multiplicative functions
1.9 Liouville's function, the divisor functions
1.10 Generalized convolutions
1.11 Bell series of Arithmetical functions

Unit II: Congruences [12 Hours]

2.1 Definition and basic properties of congruences
2.2 Residue Classes and complete residue system
2.3 Linear congruences
2.4 Reduced Residue systems and Euler-Fermat theorem
2.5 Polynomial congruences modulo p, Lagrange's Theorem and its applications
2.6 Simultaneous congruences and Chinese remainder theorem
2.7 Polynomial congruences and Prime power moduli

Unit III: Finite Abelian Groups and Their Characters [12 Hours]

3.1 Characters on finite abelian groups
3.2 The Character group
3.3 The orthogonality relation for characters
3.4 Dirichlet characters
3.5 The sum involving Dirichlet characters
3.6 The nonvanishing of L(1, χ) for real nonprincipal χ
3.7 The Dirichlet's theorem for primes of the form 4n-1 and 4n+1 (without proof)

Unit IV: Quadratic Residues and the Quadratic Reciprocity law [15 Hours]

4.1 Quadratic Residues
4.2 Legendre symbol and its properties
4.3 Evaluation of (-1/p) and (2/p)
4.4 Gauss Lemma
4.5 Quadratic reciprocity law and its applications
4.6 The Jacobi Symbol
4.7 Applications to Diophantine equations

Unit V: Introduction to Algebraic Integers [06 Hours]

5.1 Gaussian integers

5.2 Integrality

Recommended Book:

T. M. Apostol, An Introduction to Analytical Number Theory

Chapters: Chapter 2 (2.1 to 2.17) , Chapter 5 (5.1 to 5.9), Chapter 6 (6.5 to 6.10), Chapter 9 (9.1 to 9.8)

Jurgen Neukirch, Algebraic Number Theory

Chapters: Chapter 1(1.1, 1.2)

Reference Books:

1) Ivan Niven & H.S. Zuckerman, An introduction to number theory
2) David M Burton, Elementary Number Theory
3) K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory

Course Code and Title: PSMTELE-234F: Probability and Statistics

Lectures: 60 (Credits- 4)

Course Outcomes:

1. Students will able to understand probability.
2. To learn random variables, probability distributions and concept of mathematical expectation.
3. To learn Some Discrete Probability Distributions and Continuous Probability Distributions.
4. To understand Linear Regression and Correlation.

Unit I: Introduction to Probability [5 Hours]

1.1 Sample space, events, probability of an event, additive rules and conditional probability

1.2 Multiplicative rule, Bayes' rule

Unit II: Random Variable [14 Hours]

2.1 Concept of a random variable, discrete probability distribution, continuous probability distribution, joint probability distribution

2.2 Independent random variables, Chebyshev's theorem, Mean of a random variables, Variance and Covariance, Mean and Covariance of linear combinations of random variables, Functions of random variables, transformations of variables

2.3 Moments and Moment Generating Functions, definition of Expectation, theorems on Expectation and its related problems, Variance in terms of Expectation and related problems

2.4 Covariance in terms of Expectation and related problems, Variance of a Linear Combination

Unit III: Some Discrete Probability Distributions [12 Hours]

3.1 Binomial and Multinomial distributions, Hypergeometric distribution, Negative Binomial distribution, Geometric distribution

3.2 Poisson distribution and Poisson process

Unit IV: Some Continuous Probability Distributions [13 Hours]

4.1 Continuous Uniform distribution, Normal distribution, area under the normal curve, Applications of the Normal distribution, normal approximation to the Binomial Distribution

4.2 Gamma distribution, Exponential distribution, Chi-squared distribution

Unit V: Linear Regression and Correlation [14 Hours]

5.1 Simple Linear Regression, lines of Regression, Least Squares and Fitted Model

5.2 properties of Least Squares Estimators, Regression Coefficients, choice of regression model, data plot and transformations

5.3 Karl Pearson‟s Coefficient of Correlation, Properties and Problems, Spearmen's Rank Correlation Coefficient, Method of Concurrent Deviations

Unit VI: Introduction to Markov chains and Random Walks [2 Hours]

6.1 Construction of Markov chain and examples

Recommended Book:

1) Probability and Statistics for Engineers and Scientists, by R. Walpole, R.H. Myers, S.L. Myers and K. Ye (Seventh Edition, Pearson India)

Unit 1: Chapter 2

Unit 2: Chapter 3 and chapter 4

Unit 3: Chapter 5

Unit 4: Chapter 6:6.1-6.8

Unit 5: Chapter 11

Ursula Porod, Dynamics of Markov Chains for Undergraduates

Chapter: 1.1 to 1.5

Reference Books:

1) Introduction to Probability and Statistics for Engineers and Scientists, by Sheldon M. Ross (Fourth Edition)

2) A first course in Probability, by Sheldon M. Ross (Nineth Edition)

3) Mathematical Statistics, by Parimal Mukhopadhyay

4) Statistics for the Life Sciences, by M. Samules, J. Witmer and A. Schaffner (Fifth Edition, Pearson India)

5) Probability and Statistics for Engineers, by Richard Gupta, C B Gupta

SEMESTER- IV

Course Code and Title: PSMT-241: Numerical Analysis

Lectures: 60 (Credits- 4)

Course Outcomes:

1. To learn different methods to find roots of equations.
2. To learn techniques to solve system of equations.
3. Students learn eigenvalues and eigenvectors.
4. Students are able to find integration of functions and solution of Ordinary Differential Equations.

Unit I: Root Finding Methods [10 Hours]

1.1 Convergence; Floating Point Number Systems; Floating Point Arithmetic

1.2 Fixed Point Interaction Schemes; Newton's Method; Secant Method; Accelerating Convergence

Unit II: System of Equations [14 Hours]

2.1Gaussian Elimination; Pivoting Strategies

2.2 Error Estimates and Condition Number; LU decomposition; Direct Factorization

2.3 Iterative Techniques for Linear Systems: Basic Concepts and Methods

2.4 Nonlinear Systems of Equations

Unit III: Eigenvalues and Eigenvectors [10 Hours]

3.1 The Power Method

3.2 The Inverse Power Method

3.3 Reduction to Symmetric Tridiagonal Form

3.4 Eigenvalues of Symmetric Tridiagonal Matrices

Unit IV: Differentiation and Integration [14 Hours]

4.1 Numerical Differentiation, Part II

4.2 Numerical Integration – The Basics and Newton-Cotes Quadrature; Composite Newton-Cotes Quadrature

Unit V: Initial Value Problems of Ordinary Differential Equations [12 Hours]

5.1 Euler's Method; Higher-Order One-Step Methods: Taylor Methods

5.2 Runge-Kutta Methods

5.3 Multistep Methods

5.4 Convergence and Stability Analysis.

Recommended Book:

Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Prentice Hall 2007

Unit I: Chapter 1:1.2 – 1.4, Chapter 2: 2.3 – 2.6
Unit II: Chapter 3: 3.1, 3.2, 3.5, 3.6, 3.8, 3.10
Unit III: Chapter 4: 4.1, 4.2, 4.4, 4.5
Unit IV: Chapter 6: 6.2, 6.4
Unit V: Chapter 6: 6.5, 7.2-7.6

Steven C. Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientists
Chapters: MATLAB codes

Reference Books:

1) K .E. Atkinson, An Introduction to Numerical Analysis, Second Edition, John Wiley & Sons
2) John H. Mathews, Kurtis D. Fink, Numerical Methods Using Matlab, 4th Edition, Pearson Education (Singapore) Pvt. Ltd., Indian Branch, Delhi 2005
3) Numerical Analysis (9th Edition) - Richard L. Burden and J. Douglas Faires. (Brooks/Cole)
4) S. S. Sastry - Introductory methods of numerical analysis (2012, PHI Learning Pvt. Ltd.)

Course Code and Title: PSMT-242: Functional Analysis

Lectures: 60 (Credits- 4)

Course Outcomes:

1. To learn three basic principle of bounded linear map namely open mapping theorem Closed graph theorem, uniform boundedness principle.
2. To learn application of fundamental principle in functional analysis in optimization.
3. To learn weak convergence
4. To learn geometry of Hilbert spaces and normal operators on Hilbert spaces.

Unit I: Preliminaries [03 Hours]

1.1 Linear spaces and linear maps
1.2 Lebesgue measure and integration

Unit II: Fundamentals of Normed Spaces [15 Hours]

2.1 Normed spaces
2.2 Continuity of linear maps
2.3 Hahn Banach Theorems
2.4 Banach spaces

Unit III: Bounded Linear Maps on Banach Spaces [12 Hours]

3.1 Uniform boundedness Principle
3.2 Closed graph and Open Mapping Theorems

3.3 Bounded Inverse Theorem
3.4 Spectrum of Bounded Operator

Unit IV: Spaces of Bounded Linear Functionals [12 Hours]

4.1 Duals and Transposes
4.2 Duals of L^p ([a, b]) and C ([a, b])
4.3 Weak and Weak * convergence
4.4 Reflexivity

Unit V: Geometry of Hilbert Spaces [10 Hours]

5.1 Inner Product Spaces
5.2 Orthonormal sets
5.3 Approximation and Optimization
5.4 Projection and Riesz Representation Theorems

Unit VI: Bounded Operators on Hilbert Spaces [08 Hours]

6.1 Bounded Operators and Adjoints
6.2 Normal Unitary and self Adjoint Operators

Recommended Book:

B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.

Chapters: Chapter 1(Sections 2, 3), Chapter 2, Chapter 3, Chapter 4, Chapter 6, Chapter 7(Sections 25, 26)

Reference Books:

1) G. F. Simmons, Introduction to Topology and Modern Analysis, Tata McGraw Hill
2) S. Kesavan, Functional Analysis, Trim series, Hindustan Book agency
3) George Bachman, Lawrence Narici, Functional Analysis, Dover Publications.
4) E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley, 1989.

Course Code and Title: PSMT-243: Differential Geometry

Lectures: 60 (Credits- 4)

Course Outcomes:

1. To develop the geometry of curves and surfaces.
2. Students are able to understood Gauss Map and its Spherical image.
3. To learn Covariant Derivative and Curvature of Surface.
4. Students are able to understood Orientation of Surfaces.

Unit I: Curves in space and in space [03 Hours]

1.1 What is a curve?
1.2 Arc-length
1.3 Reparametrization
1.4 Closed curves

1.5 Level curves versus parameterized curves

Unit II: How much does a curve curve? [04 Hours]

2.1 Curvature
2.2 Plane curves
2.3 Space curves

Unit III: Surfaces in three dimensions [05 Hours]

3.1 What is surface?
3.2 Smooth Surfaces
3.3 Smooth maps
3.4 Tangents and derivatives
3.5 Normals and Orientability

Unit IV: Examples of surfaces [6 Hours]

4.1 Level surfaces
4.2 Quadric surfaces
4.3 Ruled surfaces and surfaces of revolution
4.4 Compact surfaces
4.5 Triply orthogonal systems
4.6 Applications of inverse function theorem

Unit V: The first fundamental form [08 Hours]

5.1 Lengths of curves on surfaces
5.2 Isometries of surfaces
5.3 Conformal mappings of surfaces
5.4 Equiareal maps and a theorem of Archimedes
5.5 Spherical Geometry

Unit VI: Curvature of Surfaces [08 Hours]

6.1 The second fundamental form
6.2 The Gauss and Weingarten maps
6.3 Normal and geodesic curvatures
6.4 Parallel transport and covariant derivative

Unit VII: Gaussian, mean and principal curvatures [10 Hours]

7.1 Gaussian and mean curvatures
7.2 Principal curvatures of a surface
7.3 Surfaces of constant Gaussian curvature
7.4 Flat surfaces
7.5 Surfaces of constant mean curvature
7.6 Gaussian curvature of compact surfaces

Unit VIII: Geodesics [08 Hours]

8.1 Definition and basic properties
8.2 Geodesic equations
8.3 Geodesics on surfaces of revolution

8.4 Geodesics as shortest paths
8.5 Geodesic co-ordinates

Unit IX: Gauss Theorem [08 Hours]

9.1 The Gauss and Codazzi –Mainardi equations
9.2 Gauss's Remarkable Theorem
9.3 Surfaces of constant Gaussian curvature
9.4 Geodesic mappings

Recommended Books:

1) Andrew Pressley, Elementary Differential Geometry, Springer Chapters: 1, 2, 4 to 10.
2) J. A. Thorpe, Elementary Topics in Differential Geometry, First Indian Reprint, Springer Publication

Reference Books:

1) Erwin Kryszig, Differential Geometry, Dover Publications Inc.
2) Christian Bar, Elementary Differential Geometry, Cambridge University Press
3) T. J. Willmore, An Introduction to Differential Geometry, Dover Publications Inc.

Course Code and Title: PSMTELE-244A - Combinatorics

Lectures: 60 (Credits- 4)

Course Outcomes:

1. To learn general counting methods for arrangements and selections.
2. Students will able to use generating functions and recurrence relations to solve problems.
3. Understand inclusion-exclusion with Venn diagrams and inclusion – exclusion formula.
4. To learn restricted positions and Rook polynomials.

Unit I: General Counting Methods for Arrangements and Selections [20 Hours]

Counting principles, arrangements and selections, arrangements and selection with repetition, distributions, binomial identities

Unit II: Generating function [14 Hours]

Generating function models, calculating coefficients of generating functions, Partitions, exponential generating functions, a summation method

Unit III: Recurrence Relations [14 Hours]

Recurrence relation models, Divide and conquer relations, solution of linear

homogeneous and Inhomogeneous recurrence relation, solution with generating functions

Unit IV: Inclusion-exclusion [12 Hours]

Counting with Venn diagrams, inclusion – exclusion formula, restricted positions and Rook polynomials, Derangements, Counting onto functions.

Recommended Book:

Alan Tucker, Applied Combinatorics (fourth edition), John Wiley & sons, New York (1995)

Unit 1: Chapter 5 (sections 1-6)
Unit 2: Chapter 6 (sections 1-5)
Unit 3: Chapter 7 (sections 1-5)
Unit 4: Chapter 8 (sections 1-3)

Reference books:

1. V. Krishnamurthy, Combinatorial, Theory and Applications, East West Press, New Delhi (1989) Scientific (1996)
2. K.D. Joshi: Foundations of discrete mathematics, Wiley
3. Marshall Hall: Combinatorial theory, Wiley

Course Code and Title: PSMTELE-244B: Machine Learning With Python

Course Outcomes:

1. Study Simple linear Regression, Multiple Regression, Logistic Regression and Decision trees.
2. Understand the Neural Networks and deep learning.
3. Students will able to learn Clustering, Natural Language processing and Network Analysis.

Unit I: Machine Learning [5 Hours]

What is Machine learning?, Overfitting and Underfitting**,** Correctness, The Bais – variance trade-off, feature extraction and selection.

Unit II: Support vector Machine theory [5 Hours]

Hyperplanes in R^n, convex functions, Classical optimization, Kernel functions

Unit III: Graph Theory [5 Hours]

Trees, Weighted graphs, Directed graph, Dijkstra Algorithm, Kruskal's algorithm

Unit IV: Hilbert spaces and approximations [5 Hours]

Hilbert spaces, Fourier Transform, Convolution, Nonlinear Function approximation,

Unit V: Tensor Products and Dynamical Systems [5 Hours]

The Tensor products, Dynamic Mode decomposition,
Linear dynamical systems and stability

Unit VII: Principal Component analysis [5 Hours]

Orthogonal transformations, Rayleigh quotient, robust and L^1 norm.

Recommended Book:

Joel Grus, Data Science from Scratch, First Principles with Python, O'Reilly Publication, Second Edition
Part I: Chapter 14 to 22

Reference Books:

1. Dirk P. Kroese, Zdravko I. Botev, Thomas Taimre, RadislavVaisman, Data Science and Machine Learning, Mathematical and Statistical Methods
2. Francois Chollet, Deep Learning with Python, Manning Publications
3. Davy Cielen, Arno D. B. Meysman, Mohmed Ali, Introducing Data Science Big Data, Machine Learning And more using Python Tools, Manning Publications

Course Code and Title: PSMTP-244B: Practical Based On Machine Learning Using Python

(Credits- 2)

Practical I: Decision tree -I
Practical II: Decision tree-II
Practical III: Constructing a simple Neural Networks (A perceptron)
Practical IV: Use of activation function (Sigmoid function, RELU, step functions)
Practical V: Training of Neural Networks using example like Fizz-Buzz
Practical VI: Creating tensors with different shapes
Practical VII: Training of Linear Layer Neural Networks.
Practical VIII: Training of Multilayer Neural Networks using Back propagation -I
Practical IX: Training of Multilayer Neural Networks using Back propagation-II
Practical X: Loss and Optimization in Neural Networks.
Practical XI: Natural language processing -I
Practical XII: Natural language processing II

One practical 4 hrs. per week

Course Code and Title: PSMTELE-244C: Dynamical Systems

Course Outcomes:

1. To learn one dimensional and two dimensional differential system.

2. To learn qualitative properties of solution of differential system by plotting phase portrait.
3. To learn how to find solution of linear differential system using exponential method.
4. To learn about stability of solutions of differential system.
5. To learn linearization of differential systems and Hartman-Grobman Theorem.
6. To learn about stable, unstable, centre manifolds of an autonomous systems.
7. To learn index theory, Poincare Bendixson theorem, Poincare sphere.
8. To learn about limit cycles of autonomous systems.

Unit I: Introduction [05 Hours]

1.1 Modelling
1.2 What Are Differential Equations?
1.3 One-Dimensional Dynamics
Population Dynamics, Mechanical Systems Oscillating Circuits, Fluid Mixing
1.5 Two-Dimensional Dynamics, Null clines, Phase Curves
1.6 The Lorenz Model
1.7 Quadratic ODEs: The Simplest Chaotic Systems

Unit II: Linear System [13 Hours]

2.1 Matrix ODE's, Eigen values and Eigenvector, Diagonalization
2.2 Two Dimensional Linear System
2.3 Exponential of Operators.
2.4 Fundamental Solution Theorem
2.5 Complex Eigen values
2.6 Multiple Eigen values, Semisimple-Nilpotent Decomposition
The Exponential, Alternatives Method
2.7 Linear Stability
2.8 Nonautonomous Linear System and Floquet Theory

Unit III: Existence and Uniqueness [05 Hours]

3.1 Existence and Uniqueness Theorem
3.2 Dependence on Initial Conditions and Parameters
3.3 Maximal Interval of Existence

Unit IV: Dynamical Systems [15 Hours]

4.1 Definitions
4.2 Flows
4.3 Global Existence of Solutions
4.4 Linearization
4.5 Stability
4.6 Lyapunov Functions
4.7 Topological Conjugacy and Equivalence
4.8 Hartman–Grobman Theorem
4.9 Omega-Limit Sets

4.10 Attractors and Basins
4.11 Stability of Periodic Orbits
4.12 Poincaré Maps

Unit V: Invariant Manifolds [10 Hours]

5.1 Stable and Unstable Sets
5.2 Heteroclinic Orbits
5.3 Stable Manifolds
5.4 Local Stable Manifold Theorem
5.5 Global Stable Manifolds
5.6 Centre Manifolds

Unit VI: The Phase Plane [12 Hours]

6.1 Nonhyperbolic Equilibria in the Plane
6.2 Two Zero Eigen values and Nonhyperbolic Nodes
6.3 Imaginary Eigen values: Topological Centers
6.4 Symmetries and Reversors
6.5 Index Theory
6.6 Poincaré–Bendixson Theorem
6.7 Liénard Systems
6.8 Behaviour at Infinity: The Poincaré Sphere

Recommended Book:

James D. Meiss, Differential Dynamical Systems, Society for Industrial and Applied Mathematics Philadelphia Publication

Reference Books:

1) Morris W. Hirsh, Stephen Smale, Robert L. Devaney, Differential Equations, Dynamical Systems and an Introduction to Chaos, Academic Press.
2) Lawrence Perko, Differential Equations and Dynamical Systems, Springer Publication

Course Code and Title: PSMTELE-244D: Commutative Algebra

Lectures: 60 (Credits-4)

Course Outcomes:

1. To learn Rings and modules.
2. To learn Tensor product, Noetherian rings
3. Students learn discrete valuations rings and Dedekind domains.

Unit I: Rings and Ideals [8 Hours]

1.1 Revision: Rings, Homomorphism
1.2 Nil radical and Jacobson radical
1.3 Operations on ideals
1.4 Extensions and contractions

Unit II: Module [12 Hours]

2.1 Free module
2.2 Finitely generated module
2.3 Exact Sequences
2.4 Tensor Product of modules
2.5 Exactness properties of tensor product

Unit III: Rings and modules of fractions [15 Hours]

3.1 Local Properties
3.2 Extended and contracted ideals in rings of fractions
3.3 Primary decomposition

Unit IV: Integral Dependence and Valuation [10 Hours]

4.1 Going up and Going down Theorems
4.2 Valuation rings
4.3 Chain conditions

Unit V: Noetherian rings and Dedekind Domain [10 Hours]

5.1Primary decompositions of Noetherian rings
5.2 Artinian rings
5.3 Discrete valuation rings
5.4 Dedekind Domains an introduction

Unit VI: Fundamental Theorems [05 Hours]

6.1 Fundamental theorem of algebra
6.1 Hilbert Nullstellensatz
6.2 Hilbert Basis Theorem

Recommended Book:

1) M. F. Atiyah, I. G. MacDonald - Introduction to Commutative Algebra (1969). Chapters 1 to 9

Reference Books:

1) N. S. Gopalkrishan : Commutative Algebra
2) Singh B. - Basic Commutative Algebra (2011, WS)
3) I. Kaplansky – Commutative Rings

Course Code and Title: PSMTELE-244E: Representation Theory Of Finite Groups

Lectures: 60 (Credits- 4)

Course Outcomes:

1. Students learn Schur's lemma, general linear group.

2. To learn representation of groups.
3. Students learn Fourier transformations

Unit I: Basics of Group Theory and Linear Algebra [8 Hours]

1.1 Group actions, General linear group, basic definitions and examples of group actions and orbits under group actions

1.2 General linear group

Unit II: Group Representations [12 Hours]

2.1 Irreducible and Indecomposable representations

2.2 Maschke's theorem and complete, reducibility, Schur's lemma

Unit III: Character Theory and Orthogonality relations [25 Hours]

3.1 Orthogonality relations, characters and class functions

3.2 The regular representation, permutation representation, representations of Abelian groups

Unit IV: Fourier Analysis on Finite Groups [15 Hours]

4.1 Periodic functions and Fourier transform

4.2 Convolutions, Fourier Inversion

4.3 Dual group

Recommended Book:

Benjamin Steinberg, Representation Theory of Finite Groups

Unit I: 2.1 to 2.3

Unit II: 3.1, 3.2

Unit III: 4.1to 4.5

Unit IV: 5.1 to 5.3

Audrey Terras, Zeta functions of Graphs, Cambridge University Press

Chapter: 18

Reference Books:

1. J. P. Serre, Linear Representations of Groups
2. James Leibeck, Representation Theory, Michael Artin, Algebra

Course Code and Title: PSMTELE-244F: Coding Theory

Lectures: 60 (Credits- 4)

Course Outcomes:

1. To learn error detection and linear codes.
2. To learn bounds in coding theory, cyclic codes.
3. Students learn decoding and error correction.

Unit I: Error detection [12 Hours]

1.1 Correction and decoding: Communication channels, Maximum likelihood Decoding

1.2 Hamming distance, Nearest neighbor / minimum distance decoding, Distance of a code

Unit II: Linear codes [18 Hours]

2.1 Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes

2.2 Generator matrix and parity check matrix, Equivalence of linear codes, Encoding With a linear code, Decoding of linear codes

2.3 Cossets, Nearest neighbor decoding for linear codes, Syndrome decoding

Unit III: Bounds in Coding Theory [16 Hours]

3.1 Main coding theory problem, lower bounds, sphere covering bounds, Gilbert Varshamov bound, Hamming Codes

3.2 Hamming bound and perfect codes Singleton bound and MDS codes

Unit IV: Cyclic codes [10 Hours]

4 .1 Definitions, Generator polynomials

4.2 Generator and parity check matrices

4.3 Decoding of cyclic codes, Burst-error-correcting codes

Unit V: Some special cyclic codes: [04 Hours]

5.1 BCH codes, Definitions, Parameters of BCH codes

Recommended Book:

San Ling and Chaoping Xing, Coding Theory- A First Course (Cambridge University Press, 2004) (Sections 2.1, to 2.5); (Sections 3.1. to 3.4 and Sections 4.1, to 4.8); (Sections 5.1, to 5.4), (7.1, to 7.4); (8.1, 8.1.1 and 8.1.2)

Reference Books:

1) Raymond Hill, A First Course in Coding Theory (Oxford)